WS 1- Geometric Series review & Tests for Convergence

1) Find the sum of the series 5 + 5/3 + 5/9 + ... + 5/243

2) Find the sum of the series 3 + 3/4 + 3/16 + 3/64 + ...

3) Does .4 + .04 + .004 +... converge or diverge? Explain why.

4) Does 2 + 2(5) + 2(25) + 2(125) ... converge or diverge? Explain why.

5) Determine whether $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt[4]{n}}$ converges.

6) Test the series $\displaystyle\sum \frac{n}{e^n}$ for convergence.

7) Determine whether $\displaystyle\sum_{n=1}^{\infty} \frac{1}{1+n^4}$ converges.

8) Determine whether $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^2+9}$ converges absolutely, converges conditionally, or diverges.

9) Determine whether $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges absolutely, converges conditionally, or diverges

10) Determine whether $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$ converges absolutely, converges conditionally, or diverges.

11) Does $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n!}$ converge or diverge?

12) Show that $\displaystyle\sum \frac{1}{n^n} = 1 + \frac{1}{2^2} + \frac{1}{3^3} + \dots$ converges.

13) Does $\displaystyle\sum \frac{n^2}{n!}$ converge or diverge?

Answer Key for WS 7

(1) $\frac{a(1-r^n)}{(1-r)} = \frac{5(1-(\frac{5}{3})^5)}{(1-\frac{5}{3})} = \boxed{88.951}$

(2) $\frac{a}{1-r} = \frac{3}{(1-\frac{3}{4})} = \boxed{12}$

(3) $a = .4 \quad r = .1$

$\boxed{\text{Converges because } |r| < 1}$

(4) $a = 2 \quad r = 5 \quad \boxed{\text{Diverges because } |r| \geq 1}$

(5) *Use Integral Test

$\int_1^a \frac{1}{x^{5/4}} = -4x^{-1/4} \Big|_1^a = -4a^{-\frac{1}{4}} + 4$

$\lim_{a\to\infty} (-4a^{-\frac{1}{4}} + 4) = 4 \quad \boxed{\text{Converges}}$

(6) *Use Integral Test

$\int_1^a \frac{x}{e^x} = -e^{-x}(1+x) \Big|_1^a = \lim_{a\to\infty} \left(-e^{-a}(1+a)\right) - \left(-\frac{1}{e}(2)\right)$

$\lim_{a\to\infty} -\frac{(1+a)}{e^a} + \frac{2}{e} = \boxed{\frac{2}{e}}$

$\boxed{\text{Converges}}$

(7) *Use Comparison Test

$\frac{1}{1+n^4} < \frac{1}{n^4}$ and $\sum \frac{1}{n^4}$ converges

so $\sum \frac{1}{1+n^4} \boxed{\text{converges}}$

(8) Alternating Series Test

a) $b_n > 0$ ✓

b) $b_n > b_{n+1}$ ✓

c) $b_n \to 0$ as $n \to \infty$

Diverges

c is not satisfied because $\frac{n^2}{n^2+9}$ approaches 1, not 0. All 3 must be satisfied to converge.

(9) *Can use comparison test or Alt Series.

Comparison

$\frac{1}{n+1} < \frac{1}{n}$ and $\frac{1}{n+1}$ converges.

Alt. Series Test

a) $b_n > 0$ ✓

b) $b_n > b_{n+1}$ ✓

c) $b_n \to 0$ as $n \to \infty$ ✓

$\boxed{\text{CONVERGES}}$

Does NOT converge absolutely - Harmonic series diverges

(10) Alt Series Test - passes all 3 so converges. Check absolute value to see if converges absolutely —

$\sum \frac{1}{n^3} \to$ converges so $\boxed{\text{converges absolutely.}}$

(11) *Use Ratio Test

$\lim_{n\to\infty} \frac{\frac{1}{(n+1)!}}{\frac{1}{n!}} = \frac{1}{(n+1)!} \cdot \frac{n!}{1} = \lim_{n\to\infty} \frac{1}{n+1} = 0 \quad \boxed{\text{CONVERGES}}$

(12) $\sum_{n=1}^{\infty} \frac{1}{n^n} = 1 + \frac{1}{2^2} + \frac{1}{3^3}$

$r = \frac{1}{2}$ so it $\boxed{\text{CONVERGES}}$

(13) $\sum \frac{n^n}{n!}$ *Use Ratio Test

$\lim_{n\to\infty} \frac{\frac{(n+1)^{n+1}}{(n+1)!}}{\frac{n^n}{n!}} = \frac{(n+1)^{(n+1)}}{(n+1)!} \cdot \frac{n!}{n^n} = \frac{(n+1)^n}{n^n} = \left(\frac{n+1}{n}\right)^n = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e$ or 2.718

$e > 1$ so $\sum \frac{n^n}{n!}$ $\boxed{\text{diverges}}$ by the Ratio Test.