ANSWER KEY FOR WS 2

① *Use Ratio Test!

$$\rho = \lim_{n\to\infty} \left| \frac{4^{n+1} x^{n+1}}{4^n x^n} \right| = |4x|$$

Set equal to 1 since $\rho<1$ converge, $\rho=1$ insufficient, $\rho>1$ diverge

$-1<|4x|<1$

$-\frac{1}{4}<x<\frac{1}{4}$ ← Interval of convergence. Plug back into equation to see open or closed brackets!

Radius of convergence $\boxed{R=\frac{1}{4}}$

$\sum_{n=0}^{\infty} 4^n(-\frac{1}{4})^n = \sum_{n=0}^{\infty}(-1)^n$ = diverges

$\sum_{n=0}^{\infty} 4^n(\frac{1}{4})^n \quad \sum_{n=0}^{\infty}(1)^n$ = diverges $|r|\geq 1$ ← geometric series test

thus, interval of convergence is $\boxed{(-\frac{1}{4},\frac{1}{4})}$

④ $\rho = \lim_{n\to\infty}\left|\frac{(x-2)^{n+1}}{(n+1)\cdot 5^{n+1}}\cdot\frac{n\cdot 5^n}{(x-2)^n}\right| = \lim_{n\to\infty}\frac{n}{n+1}\cdot\frac{5^n}{5^{n+1}}\cdot\frac{|x-2|^{n+1}}{|x-2|^n}$

$\lim_{n\to\infty}\frac{n|x-2|}{5(n+1)} = \frac{1}{5}|x-2|$ → *Check endpoints

$-1<\frac{1}{5}(x-2)<1$

$-5<(x-2)<5$

$-3<x<7$

$\sum_{n=0}^{\infty}\frac{(7-2)^n}{n\cdot 5^n} = \frac{5^n}{n5^n} = \frac{1}{n}$

Harmonic series, diverges

$\sum_{n=0}^{\infty}\frac{(-3-2)^n}{n\cdot 5^n} = \frac{(-5)^n}{n\cdot 5^n} = (-1)^n\frac{1}{n}$

Alternating Harmonic series, converges by Alt. Series Test.

Int of convergence: $-3\leq x<7$ or $[-3,7)$

because this converges at -3; diverges at 7

② $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$

$$\rho = \lim_{n\to\infty}\left|\frac{(-1)^{n+1}\frac{x^{2(n+1)}}{(2(n+1))!}}{(-1)^n\frac{x^{2n}}{(2n)!}}\right| = \left|\frac{x^{2n+2}}{(2n+2)!}\cdot\frac{2n!}{x^{2n}}\right| = \lim_{n\to\infty}\frac{x^2}{(2n+2)(2n+1)} = 0$$

*abs value tests for absolute convergence!

$\rho<1$ so it converges

$\boxed{(-\infty,\infty)}$ interval of convergence since all values of x would cause this to converge

$\boxed{R=\infty}$ ← distance from center of interval to either endpoint

⑤ $\rho: \lim_{n\to\infty}\left|\frac{2^{2n+2}x^{n+1}}{(n+1)^2}\cdot\frac{n^2}{2^{2n}x^n}\right| =$

$\lim_{n\to\infty}\frac{|x|^{n+1}}{|x|^n}\cdot\frac{n^2}{(n+1)^2}\cdot\frac{2^{2n+2}}{2^{2n}} = \lim_{n\to\infty}4|x|\frac{n^2}{(n+1)^2}$

$= 4|x|$

$-1<4x<1 \quad \boxed{R=\frac{1}{4}}$

$-\frac{1}{4}<x<\frac{1}{4}$ Plug in and test!

$\sum_{n=1}^{\infty}\frac{2^{2n}(\frac{1}{4})^n}{n^2} = \frac{2^{2n}}{4^n\cdot n^2} = \sum_{n=1}^{\infty}\frac{1}{n^2}$ = converges

p-series p>1 so

$\sum_{n=1}^{\infty}\frac{2^{2n}(-1)^n}{4^n\cdot n^2} = \sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$ ← converges by Alt series

Since both converge, interval of convergence is $[-\frac{1}{4},\frac{1}{4}]$ or $-\frac{1}{4}\leq x\leq\frac{1}{4}$

③ $$\rho = \lim_{n\to\infty}\left|\frac{\frac{x^{n+1}}{3^{n+1}}}{\frac{x^n}{3^n}}\right| = \lim_{n\to\infty}\left|\frac{x^{n+1}}{3^{n+1}}\cdot\frac{3^n}{x^n}\right| = \left|\frac{x}{3}\right|$$

$-1<\frac{x}{3}<1 \rightarrow -3<x<3 \quad \boxed{R=3}$

Plug back in to test for convergence/divergence

$\sum_{n=0}^{\infty}\frac{(-3)^n}{3^n} = (-1)^n = 1-1+1-\dots$ $|r|\geq 1$ diverges

$\sum_{n=0}^{\infty}\frac{(3)^n}{3^n} = 1^n = 1+1+1\dots$ $|r|\geq 1$ diverges

} open brackets

Interval of Convergence = $-3<x<3$, $(-3,3)$ } either is acceptable

⑥ $\rho = \lim_{n\to\infty}\left|\frac{(x+3)^{n+1}}{n+2}\cdot\frac{n+1}{(x+3)^n}\right| \lim_{n\to\infty}|x+3|\cdot\frac{n+1}{n+2} = |x+3|$

$-1<x+3<1$ Plug in points

$-4<x<-2$

$R: \frac{-4-2}{2} = \boxed{-1}$

$-4 \rightarrow \sum_{n=1}^{\infty}\frac{(-1)^n}{n+1}$ = converges by Alt series test

$-2 \rightarrow \sum_{n=1}^{\infty}\frac{1}{n+1}$ compare with $\frac{1}{n}$ $\frac{1}{n+1}<\frac{1}{n}$ and $\frac{1}{n}$ diverges so $\frac{1}{n+1}$ diverges by the comparison test

Interval of convergence

$-4\leq x<-2$

$\boxed{[-4,2)}$