# ANSWER KEY WS 2 - Part 2

⑦ Find general term $= \sum_{n=0}^{\infty} \frac{(x-1)^n}{n!}$ let $a=0$

$$\rho = \lim_{n\to\infty} \left| \frac{(x-\ )^{n+1}}{(n+1)!} \cdot \frac{n!}{(x\ )^n} \right| = \lim_{n\to\infty} = \frac{|x|\ }{(n+1)} = 0$$

Since the limit $= 0$, the series converges for all $x$. Thus the interval of convergence is $(-\infty, \infty)$ and the radius is $\infty$.

⑧ $\sum_{n=3}^{\infty} \frac{\ln n}{n} x^n$ $\quad \rho = \lim_{n\to\infty} \left| \frac{\ln(n+1)\, x^{n+1}}{n+1} \cdot \frac{n}{\ln n\, x^n} \right|$

$$\lim_{n\to\infty} = \frac{\ln(n+1)}{\ln n} \cdot \frac{n}{n+1} |x|$$

*Use L'hopital's rule to further evaluate

$$\lim_{n\to\infty} \frac{\ln(n+1)}{\ln n} = \lim_{n\to\infty} \frac{\frac{1}{n+1}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{n+1} = 1$$

Therefore $\lim_{n\to\infty} \underbrace{\frac{\ln(n+1)}{\ln n}}_{=1 \text{ by L'hopital}} \cdot \frac{n+1}{n} \cdot |x| = |x|$

$-1 < x < 1$ $\quad \boxed{R = 1}$

Plug in to series

$x = 1$ $\quad \sum_{n=3}^{\infty} \frac{\ln n}{n} \rightarrow$ use integral test

$$\int_3^a \frac{\ln x}{x}\,dx = \lim_{a\to\infty} \left( \frac{1}{2}(\ln x)^2 \right) \Big|_3^a = \lim_{a\to\infty} \frac{1}{2}\left((\ln a)^2 - (\ln 3)^2\right) = +\infty$$

DIVERGES at $x = 1$!

$x = -1$ $\quad \sum_{n=3}^{\infty} (-1)^n \frac{\ln n}{n}$ By alternating series test, series CONVERGES

$-1 \le x < 1$ so $\boxed{[-1, 1)}$

That was fun!

⑨ $\rho = \lim_{n\to\infty} \left| \frac{(x-3)^{n+1}}{2^{n+1}} \cdot \frac{2^n}{(x-3^n)} \right| = \lim_{n\to\infty} \left| \frac{x-3}{2} \right|$

$-1 < \frac{x-3}{2} < 1 \quad = \quad -2 < x-3 < 2$

$1 < x < 5$

$R = \frac{5-1}{2} = \boxed{2}$

⑩ $\rho = \lim_{n\to\infty} \left| \frac{2x^{n+2}}{n+2} \cdot \frac{n+1}{(2x)^{n+1}} \right| = |2x| \frac{n+1}{n+2}$

$\lim_{n\to\infty} = |2x|$ $\quad |2x| < 1$

$-1 < 2x < 1$

$-\frac{1}{2} < x < \frac{1}{2}$

Plug in

at $x = -\frac{1}{2}$ $\quad \sum_{n=0}^{\infty} \frac{(2(-\frac{1}{2}))^{n+1}}{n+1} = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n+1}$ converges by alt-series test

$x = \frac{1}{2}$ $\quad \sum_{n=0}^{\infty} \frac{(1)^{n+1}}{n+1} = \frac{1}{n+1}$ diverges by ~~alt series test~~ comparison test

$\frac{1}{n+1} < \frac{1}{n}$

harmonic diverges

$-\frac{1}{2} \le x < \frac{1}{2}$

$\boxed{[-\frac{1}{2}, \frac{1}{2})}$

This is an AP problem. Not so bad :)